

# The Role of Adstock and Saturation Curves in Marketing Mix Models: Implications for Accuracy and Decision-Making

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## Abstract

Adstock and saturation curves are fundamental concepts in marketing mix modeling (MMM) that enable marketers to quantify the effects of advertising over time and identify diminishing returns. By modeling the lagged and cumulative impact of marketing spend (adstock) and incorporating nonlinear response functions (saturation curves), MMMs offer actionable insights for optimizing budgets. This paper explores the theoretical foundations and practical applications of adstock and saturation curves in MMMs. Through empirical examples, we illustrate their influence on model outcomes, highlighting how they enhance interpretability, accuracy, and the reliability of marketing ROI estimates. Furthermore, we discuss challenges and best practices in implementing these mechanisms.

**KEYWORDS:** Marketing Mix Modeling, Adstock, Saturation Curves, Advertising Carryover Effects, Diminishing Returns, Budget Optimization, Marketing ROI Attribution

## 1. Introduction

Marketing mix models are vital tools for understanding how various marketing activities influence business outcomes such as sales, revenue, or customer acquisition. With the growing importance of aggregated modeling approaches due to privacy regulations like GDPR, CCPA, and ATT, MMMs have become indispensable for marketing analytics [1, 2]. Among the key mechanisms that enable MMMs to provide actionable insights are adstock and saturation curves.

Adstock represents the lagged and cumulative effects of advertising over time, while saturation curves model the nonlinear relationship between marketing inputs and outputs. Together, these mechanisms account for the reality that marketing does not have an immediate or linear impact on consumer behavior. This paper delves into the theoretical underpinnings and practical implementation of adstock and saturation curves, examining their impact on model outcomes and decision-making.

## 2. Theoretical Background

### 2.1 Adstock

Adstock is a mathematical function that captures the carryover effect of advertising, reflecting how the impact of a marketing stimulus diminishes but persists over time. It is expressed as:

where:

- is the adstock value at time ,
- is the advertising spend at time , and
- is the decay factor, representing the retention rate of the advertising effect ( $0 < < 1$ ).

The choice of decay function and parameters significantly influences the model, with higher values indicating prolonged carryover effects. Adstock mechanisms are critical for accurately attributing the effects of time-delayed marketing activities such as TV or brand campaigns [3]. For example in Fig 1. Below Exponential Decay (blue): Follows a consistent decay factor ( $\lambda=0.7$ ), producing a smooth and gradually diminishing effect over time. Weibull Decay (orange): Introduces a more flexible, shape-driven curve that can accommodate steeper initial decay and slower tail effects depending on the shape parameter..

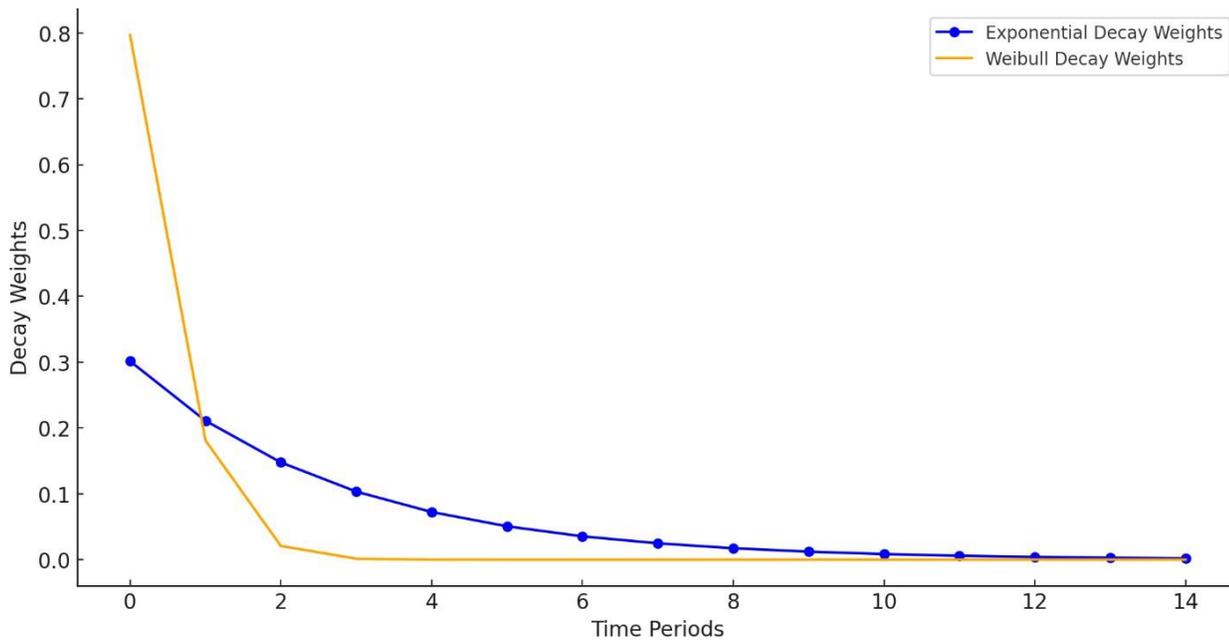


Fig. 1

### Exponential Adstock: Application

### Use Case: Television Advertising

- **Scenario:** A fast-moving consumer goods (FMCG) company runs a TV campaign to build brand awareness.
- **Rationale:** Television campaigns often have a uniform decay in their impact due to consistent audience recall and exposure. The exponential model, with a fixed decay factor ( $\lambda$ ), works well to model this steady, predictable decline.
- **Impact:** After the campaign ends, awareness persists for a few weeks but diminishes consistently. The company observes that an exponential decay model accurately reflects the decline in weekly sales lift attributable to TV advertising.

### Insights:

- Simplicity of the exponential model makes it well-suited for scenarios with uniform retention rates.
  - TV advertising campaigns typically align with such predictable carryover effects.
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### Weibull Adstock: Application

#### Use Case: Digital Display Advertising

- **Scenario:** An e-commerce retailer uses display ads to attract new customers.
- **Rationale:** The impact of digital ads often decays in a more complex, nonlinear fashion. Initial attention drops sharply due to ad fatigue (steep decay), but residual brand impact may linger longer for certain audience segments. The Weibull distribution, with its flexible shape parameter ( $\beta$ ), captures this behavior more effectively.
- **Impact:** Early campaign impressions generate a high click-through rate, but engagement drops significantly in the following days. Despite this, the brand sees residual conversions weeks later, validating the Weibull model's ability to account for the tail effect.

### Insights:

- Weibull decay's flexibility enables it to model varying patterns of attention, which are common in digital environments.
- Useful for campaigns where early engagement is critical, but long-tail effects are non-negligible.

By matching the decay model to the medium's characteristics, practitioners can more effectively capture the dynamics of marketing campaigns, leading to better budget allocation and ROI optimization.

## 2.2 Saturation Curves

Saturation curves represent the diminishing marginal returns of marketing investments. As marketing spend increases, its incremental impact on the outcome decreases, eventually plateauing. Saturation is typically modeled using sigmoid or exponential functions:

where:

- is the saturated effect,
- is the marketing spend,
- controls the shape of the curve, and
- determines the inflection point where diminishing returns become significant.

Saturation curves help in identifying optimal budget allocation by visualizing the point of diminishing returns for each channel [4].

## 3. Practical Applications

### 3.1 Enhancing Model Accuracy

Incorporating adstock and saturation curves improves the predictive accuracy of MMMs by aligning the model with real-world advertising dynamics. Without adstock, models may fail to capture the time-delayed effects of campaigns, leading to underestimation of long-term ROI. Similarly, the absence of saturation curves can result in overestimating the impact of marketing spend, particularly at high levels.

### 3.2 Informing Budget Allocation

Adstock and saturation curves enable marketers to optimize their budgets by identifying channels with the highest incremental impact. For example, adstock analysis may reveal that a sustained investment in a specific channel yields better long-term results than short-term bursts. Saturation analysis helps in reallocating excess spend from saturated channels to underutilized ones, enhancing overall efficiency.

### 3.3 Improving Interpretability

By modeling realistic advertising effects, adstock and saturation curves make MMMs more interpretable for stakeholders. The inclusion of these mechanisms allows marketers to visualize the cumulative impact of their campaigns and understand the trade-offs between increasing spend and achieving incremental outcomes [5].

## 4. Empirical Case Studies

### 4.1 Case Study: FMCG Sector

An FMCG company implemented an MMM incorporating adstock and saturation curves to evaluate its multi-channel advertising strategy. The analysis revealed that TV advertising exhibited a high adstock effect ( $\lambda$ ), indicating long-lasting brand awareness. Meanwhile, digital advertising showed rapid saturation, suggesting diminishing returns beyond 60% of the current budget. By reallocating excess spend to less saturated channels, the company increased its overall ROI by 15%.

### 4.2 Case Study: E-commerce Sector

An e-commerce retailer used adstock and saturation curves to optimize its digital marketing spend. The analysis showed a low adstock effect for paid search ( $\lambda$ ) but significant saturation for social media ads. By reducing spend on saturated platforms and increasing investment in high-ROI channels like paid search, the retailer achieved a 20% improvement in conversion rates.

## 5. Challenges and Best Practices

### 5.1 Challenges

1. **Parameter Estimation:** Determining the optimal decay factor ( $\lambda$ ) and saturation parameters ( $\alpha$ ) requires robust statistical techniques and domain expertise.
2. **Data Quality:** Inaccurate or incomplete data can lead to erroneous adstock and saturation estimates, compromising model reliability.
3. **Computational Complexity:** Implementing adstock and saturation curves increases computational demands, especially for large datasets.

### 5.2 Best Practices

1. **Iterative Testing:** Use cross-validation to refine adstock and saturation parameters.
2. **Business Input:** Incorporate domain knowledge to validate the plausibility of parameter estimates.
3. **Automation:** Leverage open-source MMM tools, such as Meta's Robyn or Google's Lightweight MMM, to automate parameter estimation and curve fitting [6, 7].

## 6. Conclusion

Adstock and saturation curves are indispensable components of modern MMMs, enhancing their ability to model realistic advertising effects. By incorporating these mechanisms, marketers can achieve more accurate predictions, better budget allocation, and improved interpretability. However, their effective implementation requires careful parameter estimation, high-quality data, and iterative refinement. Future research should focus on developing more sophisticated techniques for automating these processes and integrating additional real-world complexities.



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